Phase transitions induced by noise cross-correlations

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A general approach for treating the spatially extended stochastic systems with the nonlinear damping and correlations between additive and multiplicative noises is developed. Within the modified cumulant expansion method, we derive an effective Fokker-Planck equation with stationary solutions that describe the character of the ordered state. We find that the fluctuation cross-correlations lead to a symmetry breaking of the distribution function even in the case of zero-dimensional system. In a general case, continuous, discontinuous and reentrant noise induced phase transitions take place. It appears that the cross-correlations play the role of bias field which can induce a chain of phase transitions of different nature. Within the mean field approach, we give an intuitive explanation of the system behavior by an effective potential of the thermodynamic type. This potential is written in the form of an expansion with coefficients defined by the temperature, intensity of spatial coupling, autocorrelation and cross-correlation times and intensities of both additive and multiplicative noises.

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I. INTRODUCTION

The constructive role of the fluctuations of environment, usually called noises, is of considerable interest in studying the stochastic processes. An incomplete list of such processes includes noise induced unimodal-bimodal transitions [1], stochastic resonance [2], noise-induced spatial patterns and phase transitions [3], etc. In a general case, considering the stochastic dynamics, one should deal with a problem of taking into account the correlations between random sources. Several special methods have been developed in this direction. Most popular of them are as follows: (i) the cumulant expansion method [4,5]; (ii) the spectral width expansion method [1,6]; (iii) the unified colored noise approximation where evolution equations for both a stochastic variable and a random force are combined within a unique equation of motion [7–10].

The wide spectra of works are aimed to explore an effect of correlations of the fluctuations in extended systems (see Ref. [3], and references therein). It appears that when the system is nonlinear, the spatial coupling and noise correlations force the system to exhibit a special behavior known as reentrant phase transition [11]. Moreover, such a reentrance can be observed even in the limit of weakly correlated noise: if the noise self-correlation time is $\tau \to 0$, the system undergoes a single reentrant phase transition [12]. In the opposite limit of the strong correlated noise ($\tau \to \infty$) a chain of reentrant phase transitions can be realized [8].

A quite peculiar picture is observed in the case of the presence of several noises with cross-correlations which leads to the remarkable and counterintuitive phenomena related to a transformation of phase transition type. In such a case, the stochastic system undergoes a chain of phase transitions with the appearance of a metastable phase, whereas the bare potential does not assume the existence of such a phase [13]. Unfortunately, nowadays we have a scanty collection of works devoted to this phenomenon. Perhaps it can be explained by problems in both the natural and computer experiments realization for extended systems, on the one hand, and by the lack of the theoretical tools and methods to perform the corresponding calculations, on the other hand.

The above pointed phenomena force to reconsider the theoretical approaches describing the noise induced phase transitions in extended systems developed before. The works concerning such a problem consider specific models and there is no general description of a transformation picture of phase transition. Therefore, the problem of the fluctuation which induces rearrangement of the system behavior is an open question that should be clarified.

In this paper, we consider the general situation of presenting the cross-correlation contribution of two noises within the picture of phase transitions. We explore an extended stochastic system which obeys the archetypal model of Brownian particle. The adequate scheme to specify the statistical properties of the system with nonlinear kinetic coefficient is introduced in the overdamped limit. Within a simplest model with nonlinear damping, drift caused by Landau-like potential and two colored (multiplicative and additive) noises, we show how the system can undergo noise induced phase transitions. We find that the phase transitions of both continuous and discontinuous character are realized as biased phase transitions. Analysis of phase and bifurcation diagrams shows that the cross-correlations lead to metastable states inherent in first order phase transitions.

The paper is organized in the following manner. Section II is devoted to the development of the analytical approach to study noise induced phase transitions on the basis of kinetic equation for the probability density function and cumulant expansion method.

In Sec. III, we apply the derived formalism to the noise induced phase transitions in the simplest case of the Ginzburg-Landau model with both multiplicative and additive noises and kinetic coefficient which depends on the stochastic variable. On the basis of the obtained drift and diffusion coefficients we build a multiplicative stochastic process. The related probability distribution function combined with a

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self-consistency condition allow us to investigate the corresponding phase diagram and stationary behavior of the order parameter (Sec. IV). The discussion in Sec. V is based on the representation of the effective stochastic process within the framework of the mean field approach. Such a consideration allows us to study the noise induced phase transitions by analogy with the standard Landau scheme. It appears that the noise cross-correlations and a spatially dependent damping coefficient lead to the transformation of the continuous phase transition into a discontinuous one. Finally, the main results and perspectives of the work are collected in the conclusion (Sec. VI).

II. MAIN RELATIONS

Consider a Brownian particle under influence of an effective potential $\mathcal{F}[x(\mathbf{r})]$ and a damping characterized by the viscosity coefficient $\gamma(x)$. The generalized equations for the time evolution of the d-dimensional scalar field x and the conjugate momentum p read

$$m\dot{x} = p, \tag{1}$$

$$\dot{p} + \frac{\gamma(x)}{m}p = -\frac{\delta\mathcal{F}}{\delta x(\mathbf{r},t)} + g_\mu(x)\zeta_\mu(\mathbf{r},t) \tag{2}$$

where m is the effective particle mass, an overdot stands for the derivative with respect to the time t, $\mathbf{r}$ is the space coordinate; the effective potential is reduced to the Ginzburg-Landau form [14]

$$\mathcal{F} = \int \left[V_0(x) + \frac{D}{4d}|\boldsymbol{\nabla} x|^2 \right] d\mathbf{r} \tag{3}$$

with $V_0(x)$ and $D > 0$ being a specific thermodynamic potential and an inhomogeneity constant $\nabla \equiv \partial/\partial\mathbf{r}$. The last term in Eq. (2), where index μ numerates different noises to be summarized in accordance with the Einstein rule, represents Langevin forces which act with amplitudes $g_\mu(x)$ and stochastically alternating functions ζ_μ. Neglecting a space correlations, we focus on time correlations between forces ζ_μ, i.e.,

$$\langle \zeta_\mu(\mathbf{r},t)\zeta_\nu(\mathbf{r}',t')\rangle = \delta(\mathbf{r}-\mathbf{r}')C_{\mu\nu}(t-t'). \tag{4}$$

Inserting Eq. (2) into the result of differentiation of Eq. (1) over time, we represent the evolution equation for the quantity x in the form

$$m\ddot{x} + \gamma(x)\dot{x} = f(x) + \frac{D}{2d}\Delta x + g_\mu(x)\zeta_\mu, \tag{5}$$

where $f(x) = -dV_0/dx$ is a deterministic force.

To study statistical properties of the system one needs to find the probability density function $P = P(p,x,t)$ of the system states distribution in the phase space $\{x,p\}$. To this end, we represent the system on the regular d-dimensional lattice with mesh size ℓ where $\gamma_i = \gamma(x_i)$, $f_i = f(x_i)$, $g_{\mu i} = g_\mu(x_i)$. Then, the differential equation with partial derivatives (5) is reduced to the usual differential equation

$$m\ddot{x}_i + \gamma_i\dot{x}_i = f_i + \frac{D}{2d\ell^2}\sum_j \hat{D}_{ij}x_j + g_{\mu i}\zeta_{\mu i} \tag{6}$$

due to representation of the Laplacian operator on a grid as follows:

$$\Delta \to \frac{1}{\ell^2}\Delta_i,$$

$$\Delta_i \equiv \sum_j \hat{D}_{ij} = \sum_j (\delta_{NN(i),j} - 2d\delta_{ij}), \tag{7}$$

where $NN(i)$ denotes a set of the nearest neighbors of the site i, whose number $2d$ is fixed by the lattice dimension d.

By definition, the probability density function is given by the averaging of the density function $\rho(x_i,p_i,t)$ of the microscopic states distribution in the phase space over noises:

$$P(x_i,p_i,t) = \langle \rho(x_i,p_i,t)\rangle. \tag{8}$$

To construct an equation for the macroscopic density function $P = P(x_i,p_i,t)$ we exploit the conventional device to proceed from the continuity equation to the microscopic one $\rho = \rho(x_i,p_i,t)$:

$$\frac{\partial\rho}{\partial t} + \left[\frac{\partial}{\partial x_i}(\dot{x}_i\rho) + \frac{\partial}{\partial p_i}(\dot{p}_i\rho) \right] = 0. \tag{9}$$

Inserting the time derivative of the momentum $\dot{p} = m\ddot{x}$ from Eq. (6) into Eq. (9), we obtain

$$\frac{\partial\rho}{\partial t} = (\hat{\mathcal{L}} + \hat{\mathcal{N}}_\mu\zeta_\mu)\rho, \tag{10}$$

where the operators $\hat{\mathcal{L}}$ and $\hat{\mathcal{N}}_\mu$ are defined as follows:

$$\hat{\mathcal{L}} \equiv -\frac{p_i}{m}\frac{\partial}{\partial x_i} - \frac{\partial}{\partial p_i}\left(f_i + \frac{D}{2d\ell^2}\sum_j \hat{D}_{ij}x_j - \frac{\gamma_i}{m}p_i \right), \tag{11}$$

$$\hat{\mathcal{N}}_\mu \equiv -g_{\mu i}\frac{\partial}{\partial p_i}. \tag{12}$$

Within the interaction representation, the microstate density function reads

$$\wp = e^{-\hat{\mathcal{L}}t}\rho \tag{13}$$

to reduce Eq. (10) to the form

$$\frac{\partial\wp}{\partial t} = \sum_\mu \hat{\mathcal{R}}_\mu\wp, \tag{14}$$

$$\hat{\mathcal{R}}_\mu = \hat{\mathcal{R}}_\mu(x_i,p_i,t) \equiv \zeta_\mu(e^{-\hat{\mathcal{L}}t}\hat{\mathcal{N}}_\mu e^{\hat{\mathcal{L}}t}). \tag{15}$$

The well-known cumulant expansion method [4] serves as a standard and effective device to solve such a type of stochastic equation. Neglecting terms of the order $O(\hat{\mathcal{R}}_\mu^3)$, we get the kinetic equation in the form

$$\frac{\partial}{\partial t}\langle\wp\rangle(t)=\left[\sum_{\mu\nu}\int_0^t\langle\hat{\mathcal{R}}_\mu(t)\hat{\mathcal{R}}_\nu(t')\rangle \mathrm{d}t'\right]\langle\wp\rangle(t). \quad (16)$$

Within the original representation, the equation for the probability density (8) reads

$$\frac{\partial}{\partial t}P(t)=\left\{\hat{\mathcal{L}}+\int_0^t C_{\mu\nu}(\tau)[\hat{\mathcal{N}}_\mu(e^{\hat{\mathcal{L}}\tau}\hat{\mathcal{N}}_\nu e^{-\hat{\mathcal{L}}\tau})]\mathrm{d}\tau\right\}P(t). \quad (17)$$

If the physical time is much larger than a correlation scale ($t\gg\tau_\mu$), we can replace the upper limit of the integration by $t=\infty$. Then, expanding exponents, we obtain for the perturbation expansion

$$\frac{\partial P}{\partial t}=(\hat{\mathcal{L}}+\hat{\mathcal{C}})P, \quad (18)$$

where collision operator

$$\hat{\mathcal{C}}\equiv\sum_{n=0}^{\infty}\hat{\mathcal{C}}^{(n)},\quad \hat{\mathcal{C}}^{(n)}\equiv M_{\mu\nu}^{(n)}(\hat{\mathcal{N}}_\mu\hat{\mathcal{L}}_\nu^{(n)}) \quad (19)$$

is determined through the commutators

$$\hat{\mathcal{L}}_\nu^{(n+1)}=[\hat{\mathcal{L}},\hat{\mathcal{L}}_\nu^{(n)}],\quad \hat{\mathcal{L}}_\nu^{(0)}\equiv\hat{\mathcal{N}}_\nu \quad (20)$$

and moments of the correlation function

$$M_{\mu\nu}^{(n)}=\frac{1}{n!}\int_0^\infty \tau^n C_{\mu\nu}(\tau)d\tau. \quad (21)$$

To perform the following calculations we shall restrict ourselves to considering overdamped systems where the variation scales t_s, ℓ, x_s, v_s, γ_s, f_s, D_s, and g_s of the time t, the coordinate $\mathbf{r}$, the quantity x, the velocity $v\equiv p/m$, the damping coefficient $\gamma(x)$, the force $f(x)$, the coupling constant D, and the noise amplitudes $g_\mu(x)$, respectively, obey the following conditions:

$$\frac{v_st_s}{x_s}\equiv\epsilon^{-1}\gg1,\quad \frac{\gamma_st_s}{m}\equiv\epsilon^{-2}\gg1,$$

$$\frac{f_st_s}{v_sm}=\frac{g_st_s}{v_sm}\equiv\epsilon^{-1}\gg1,\quad \frac{D_sx_st_s}{mv_s\ell^2}=\epsilon^{-1}\gg1. \quad (22)$$

These conditions means a hierarchy of the damping and the deterministic/stochastic forces are characterized by relations

$$\frac{f_s}{\gamma_sv_s}=\frac{g_s}{\gamma_sv_s}\equiv\epsilon\ll1,\quad \frac{D_sx_s}{\gamma_sv_s\ell^2}\equiv\epsilon\ll1. \quad (23)$$

As a result, the dimensionless system of equations (1) and (2) takes on the discretized form

$$\frac{\partial x_i}{\partial t}=\epsilon^{-1}v_i,$$

$$\frac{\partial v_i}{\partial t}=-\epsilon^{-2}\gamma_iv_i+\epsilon^{-1}\left[f_i+\frac{D}{2d}\sum_j\hat{D}_{ij}x_j+g_{\mu i}\zeta_{\mu i}(t)\right]. \quad (24)$$

Respectively, the Fokker-Planck equation (18) reads

$$\left(\frac{\partial}{\partial t}-\hat{\mathcal{L}}\right)P=\epsilon^{-2}\hat{\mathcal{C}}P, \quad (25)$$

where the operator

$$\hat{\mathcal{L}}\equiv\epsilon^{-1}\hat{\mathcal{L}}_1+\epsilon^{-2}\hat{\mathcal{L}}_2 \quad (26)$$

has the components

$$\hat{\mathcal{L}}_1\equiv-v_i\frac{\partial}{\partial x_i}-\left(f_i+\frac{D}{2d}\sum_j\hat{D}_{ij}x_j\right)\frac{\partial}{\partial v_i},$$

$$\hat{\mathcal{L}}_2\equiv\gamma_i\frac{\partial}{\partial v_i}v_i. \quad (27)$$

The collision operator is defined by expressions such as Eqs. (19)–(21):

$$\epsilon^{-2}\hat{\mathcal{C}}=\sum_{n=0}^{\infty}\hat{\mathcal{C}}^{(n)},\quad \hat{\mathcal{C}}^{(n)}=M_{\mu\nu}^{(n)}(\hat{\mathcal{N}}_\mu\hat{\mathcal{L}}_\nu^{(n)}),$$

$$\hat{\mathcal{L}}_\nu^{(0)}=\epsilon^{-2}\hat{\mathcal{N}}_\nu,\quad \hat{\mathcal{N}}_\mu\equiv-g_{\mu i}\frac{\partial}{\partial v_i}. \quad (28)$$

After suppressing the factor ϵ^{-2}, the collision operator written with accuracy up to the first order in $\epsilon\ll1$ takes on the explicit form

$$\hat{\mathcal{C}}=(M_{\mu\nu}^{(0)}-\gamma_iM_{\mu\nu}^{(1)})g_{\mu i}g_{\nu i}\frac{\partial^2}{\partial v_i^2}+\epsilon M_{\mu\nu}^{(1)}g_{\mu i}g_{\nu i}\times\left[\frac{\partial^2}{\partial x_i\,\partial v_i}-\frac{1}{g_{\nu i}}\left(\frac{\partial g_{\nu i}}{\partial x_i}\right)\left(\frac{\partial}{\partial v_i}+v_i\frac{\partial^2}{\partial v_i^2}\right)\right]+O(\epsilon^2). \quad (29)$$

To obtain the usual probability function $\mathcal{P}(x_i,t)$ we consider velocity moments of the initial distribution function $P(x_i,v_i,t)$ in the standard form [6]

$$\mathcal{P}_n(x_i,t)\equiv\int v_i^nP(x_i,v_i,t)dv_i, \quad (30)$$

where integration over all set $\{v_i\}$ is performed. Then, multiplying the Fokker-Planck equation (25) by the factor v^n and integrating over velocities, one obtains the following recurrent relations:

$$\epsilon^2\frac{\partial\mathcal{P}_n}{\partial t}+n\gamma_i\mathcal{P}_n+\epsilon\left[\frac{\partial\mathcal{P}_{n+1}}{\partial x_i}-n\left(f_i+\frac{D}{2d}\sum_j\hat{D}_{ij}x_j\right)\mathcal{P}_{n-1}\right]=n(n-1)(M_{\mu\nu}^{(0)}-\gamma_iM_{\mu\nu}^{(1)})g_{\mu i}g_{\nu i}\mathcal{P}_{n-2}-\epsilon nM_{\mu\nu}^{(1)}\times\left[g_{\mu i}g_{\nu i}\frac{\partial\mathcal{P}_{n-1}}{\partial x_i}+ng_{\mu i}\left(\frac{\partial g_{\nu i}}{\partial x_i}\right)\mathcal{P}_{n-1}\right]+O(\epsilon^2). \quad (31)$$

At $n=0$, we obtain the equation for the distribution function $\mathcal{P}\equiv\mathcal{P}_0(x_i,t)$:

$$\frac{\partial\mathcal{P}}{\partial t} = -\epsilon^{-1}\frac{\partial\mathcal{P}_1}{\partial x_i}. \tag{32}$$

The expression for the first moment $\mathcal{P}_1$ follows from Eq. (31) where $n=1$ and only terms of the first order in ϵ are kept:

$$\mathcal{P}_1 = \frac{\epsilon}{\gamma_i}\left\{\left(f_i + \frac{D}{2d}\sum_j \hat{D}_{ij}x_j\right)\mathcal{P} - \frac{\partial\mathcal{P}_2}{\partial x_i} - M^{(1)}_{\mu\nu}\times\left[g_{\mu i}g_{\nu i}\frac{\partial\mathcal{P}}{\partial x_i} + g_{\mu i}\left(\frac{\partial g_{\nu i}}{\partial x_i}\right)\mathcal{P}\right]\right\}. \tag{33}$$

The second moment $\mathcal{P}_2$ can be obtained if one puts $n=2$ in Eq. (31) and takes into account only terms of zeroth order over the parameter $\epsilon\ll 1$:

$$\mathcal{P}_2 = \left(\frac{M^{(0)}_{\mu\nu}}{\gamma_i} - M^{(1)}_{\mu\nu}\right)g_{\mu i}g_{\nu i}\mathcal{P}. \tag{34}$$

Inserting Eq. (34) into Eq. (33) and the result into Eq. (32) we obtain the Fokker-Planck equation in the Kramers-Moyal form

$$\frac{\partial\mathcal{P}}{\partial t} = -\frac{\partial}{\partial x_i}(\mathcal{D}_1\mathcal{P}) + \frac{\partial^2}{\partial x_i^2}(\mathcal{D}_2\mathcal{P}), \tag{35}$$

where effective drift and diffusion coefficients are as follows:

$$\mathcal{D}_1 = \frac{1}{\gamma_i}\left\{\left(f_i + \frac{D}{2d}\sum_j \hat{D}_{ij}x_j\right) + \left[M^{(0)}_{\mu\nu}g_{\mu i}g_{\nu i}\frac{\partial\gamma_i^{-1}}{\partial x_i} + M^{(1)}_{\mu\nu}g_{\mu i}\left(\frac{\partial g_{\nu i}}{\partial x_i}\right)\right]\right\}, \tag{36}$$

$$\mathcal{D}_2 = \frac{M^{(0)}_{\mu\nu}}{\gamma_i^2}g_{\mu i}g_{\nu i}. \tag{37}$$

To proceed, one needs to pass from the grid representation to a continuous one. Doing so, we use the mean-field approximation to replace the second term of effective interaction force in Eq. (36):

$$\sum_j \hat{D}_{ij}x_j \equiv \left(\sum_{NN(i)} x_{NN(i)} - 2dx_i\right) \to 2d(\eta - x), \tag{38}$$

where $NN(i)$ are nearest neighbors of the site i, an order parameter $\eta\equiv\langle x\rangle$ is defined through the self-consistency equation

$$\eta = \int_{-\infty}^{\infty} x\mathcal{P}_\eta(x)dx, \tag{39}$$

and $\mathcal{P}_\eta(x)$ is a solution of the Fokker-Planck equation (35). Under stationary condition, the relevant distribution function has the form

$$\mathcal{P}_\eta(x) = \frac{\mathcal{Z}_\eta^{-1}}{\mathcal{D}_2(x)}\exp\left(\int_{-\infty}^{x}\frac{\mathcal{D}_1(x',\eta)}{\mathcal{D}_2(x')}dx'\right), \tag{40}$$

where the partition function

$$\mathcal{Z}_\eta = \int_{-\infty}^{\infty}\frac{dx}{\mathcal{D}_2(x)}\exp\left(\int_{-\infty}^{x}\frac{\mathcal{D}_1(x',\eta)}{\mathcal{D}_2(x')}dx'\right) \tag{41}$$

takes care of the normalization condition. Equation (39) has solutions within the domain bounded by the Newton-Raphson condition

$$\int_{-\infty}^{\infty} x\frac{\partial}{\partial\eta}\mathcal{P}_\eta(x)\bigg|_{\eta=0} dx = 1 \tag{42}$$

obtained by differentiating Eq. (39) over the order parameter η.

III. MODEL OF CORRELATION BETWEEN ADDITIVE AND MULTIPLICATIVE NOISES

To apply the general results obtained in Sec. II we consider in detail the simplest model of correlated additive and multiplicative noises with amplitudes

$$g_a(x) = 1, \quad g_m(x) = \mathrm{sgn}(x)|x|^a, \tag{43}$$

where the exponent is $a\in[0,1]$ and the sign function is introduced to take into account the direction of the Langevin force. We will focus on the prototype system concerning the Ginzburg-Landau model with the potential

$$V_0(x) = -\frac{\varepsilon}{2}x^2 + \frac{1}{4}x^4, \tag{44}$$

where ε is a parameter corresponding to dimensionless temperature counted off a critical value in negative direction. In correspondence with the line of consideration [15], we take up the viscosity coefficient in the form

$$\gamma(x) = |x^2 - 1|^{-\alpha}, \tag{45}$$

where the positive index α stands to measure the damping singularity near the bare state $x=1$.

Next, we suppose the noises to be Gaussian distributed with zero mean, white in space and colored in time according to the correlation matrix

$$\hat{\mathbf{C}}(\tau) = \begin{pmatrix} \frac{\sigma_a^2}{\tau_a}e^{-|\tau|/\tau_a} & \frac{\sigma_a\sigma_m}{\tau_c}e^{-|\tau|/\tau_c} \\ \frac{\sigma_m\sigma_a}{\tau_c}e^{-|\tau|/\tau_c} & \frac{\sigma_m^2}{\tau_m}e^{-|\tau|/\tau_m} \end{pmatrix}, \tag{46}$$

where σ_a and σ_m are amplitudes of additive and multiplicative noises, respectively, τ_a and τ_m are corresponding autocorrelation times, τ_c is a time of the cross-correlation between noises. Moments (21) of both zero and first orders of correlation matrix (46) are as follows:

$$\hat{\mathbf{M}}^{(0)} = \begin{pmatrix} \sigma_a^2 & \sigma_a\sigma_m \\ \sigma_m\sigma_a & \sigma_m^2 \end{pmatrix},$$

$$\hat{\mathbf{M}}^{(1)} = \begin{pmatrix} \tau_a \sigma_a^2 & \tau_c(\sigma_a \sigma_m) \\ \tau_c(\sigma_m \sigma_a) & \tau_m \sigma_m^2 \end{pmatrix}. \tag{47}$$

Then, the expressions (36) and (37) for effective drift and diffusion coefficients take the form

$$\mathcal{D}_1 = |x^2-1|^{\alpha}\{[D(\eta - x) + x(\varepsilon - x^2)] + a\sigma_m |x|^{a-1} \times [\sigma_a \tau_c + \sigma_m \tau_m \mathrm{sgn}(x)|x|^a]\} + 2\alpha x (x^2-1)^{2\alpha-1}[\sigma_a + \sigma_m \mathrm{sgn}(x)|x|^a]^2, \tag{48}$$

$$\mathcal{D}_2 = (x^2-1)^{2\alpha}[\sigma_a + \sigma_m \mathrm{sgn}(x)|x|^a]^2. \tag{49}$$

It is worthwhile to notice at $\gamma(x)=\mathrm{const}(\alpha=0)$ the additive noise can not give a contribution to the drift coefficient (48) related to the first of moments (47).

IV. NOISE CORRELATION INDUCED TRANSITIONS

We start the consideration of the influence of noise cross-correlations on the system behavior from the self-consistency equation (39) where the stationary distribution (40) is given through the drift and diffusion coefficients (48) and (49). It is well known in the theory of phase transitions that the symmetry breaking causes the ordered state corresponding to the solution $\eta \neq 0$, while the disordered phase is related to $\eta = 0$. In the absence of the multiplicative noise, the symmetry of the stochastic distribution can be broken only by the interaction force (38) which plays the role of a conjugate field related to the order parameter η. The principal feature of far-off-equilibrium systems with colored noise is that the symmetry can be restored due to the combined effect of both the multiplicative noise and the system nonlinearity [8,12]. Therefore, a reentrant phase transition in such systems takes place. We aim to demonstrate that the picture of the phase transition can be crucially changed by means of noise cross-correlations.

First, we consider the solution of Eq. (39) at different values of the noise cross-correlation scale τ_c. As it is shown in Fig. 1, in the absence of both noises $(\sigma_a=\sigma_m=0)$ and coupling $(D=0)$ the system behaves in an usual manner, as in a square-root law (dashed curve with both vertical derivative in the point of origin and symmetry with respect to the ε axis). Such a behavior means the appearance of the maxima of the distribution (40) at the points $\pm\sqrt{\varepsilon}$ which can be interpreted as standard noise induced second order transition with mean value $\eta=0$ [16]. The situation is changed principally if we switch on the noises and coupling. First, the above symmetry is broken and only the negative value of the order parameter in the limit of small cross-correlations survives (curve 1). Combined effect of correlated noises, system nonlinearity and spatial coupling leads to the change of the order parameter sign at small values η. Indeed, as can be seen from the curves 2, 3, an increase in the cross-correlation time τ_c shifts negative solution weakly into the positive domain causing the reorientation transition at the driving parameter ε_r. In addition to this transition, an increase in the cross-correlation scale leads to the appearance of the positive solutions according to discontinuous phase transitions which

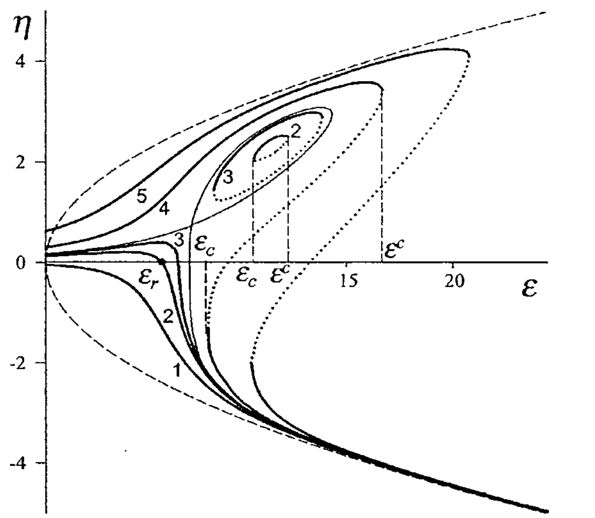

FIG. 1. Dependence of the order parameter η on the control parameter ε at $a=0.8$, $\alpha=0$, $\sigma_a^2=4.84$, $\sigma_m^2=0.01$, $\tau_m=0.01$, $D=1.0$. Curves 1, 2, 3, 4, 5 correspond to $\tau_c \to 0$, $\tau_c=2.5$, 3.0, 5.0, 10.0, respectively. The dashed curve relates to the bare dependence $\eta=\pm\sqrt{\varepsilon}$, dotted curves correspond to the unstable solutions.

has reentrant nature within domain bounded by both lower ε_c and upper ε^c boundaries **(**see elliptic form parts of curves 2, 3 where solid and dotted lines relate to (meta)stable [17] and unstable solutions**)**. With subsequent growth of the cross-correlation time above the value τ_{cr} (thin solid curve in Fig. 1) back bifurcation occurs and temperature dependence of the order parameter takes one-connected character. This means that the increase of the control parameter ε leads to the (meta)stable branch of positive magnitudes η initially (solid curve), then the unstable branch (dotted curve) follows from the point ε^c down to ε_c, and finally the negative (meta)stable state is merged. A further increase in τ_c results in the formation of a hysteresis loop in the $\eta(\varepsilon)$ dependence where both (meta)stable and unstable states appear to be solutions of Eq. (39) (see curves 4, 5). Thus, one can conclude that the reorientation phase transitions exist in systems with cross-correlated fluctuations.

In Fig. 2, we plot a phase diagram in (ε, τ_c) plane to show the influence of the noise cross-correlation scale on the bifurcation magnitudes of the control parameter. One can see that negative values of the order parameter η exist for small cross-correlation times τ_c in the whole ε domain, denoted as N in Fig. 2. An increase in τ_c leads to the reorientation phase transition where the positive solutions $\eta>0$ appears in the domain P. The line of such a transition is determined by the self-consistency equation (39) at condition $\eta=0$. At a magnitude τ_0, a doubly bounded domain R appears where a reentrant transition is realized. At the critical value τ_{cr} corresponding to the bifurcation curve in Fig. 1 the domain R passes to a region M where all stable, metastable and unstable phases take place. Over the critical correlation time $(\tau_c > \tau_{\mathrm{cr}})$, the dotted curve relates to the lower critical value ε_c of the control parameter in Fig. 1 (curve 4).

The influence of the multiplicative noise intensity σ_m^2 on the phase transition picture is demonstrated in Fig. 3. The upper panel relates to the moderate cross-correlation times τ_c

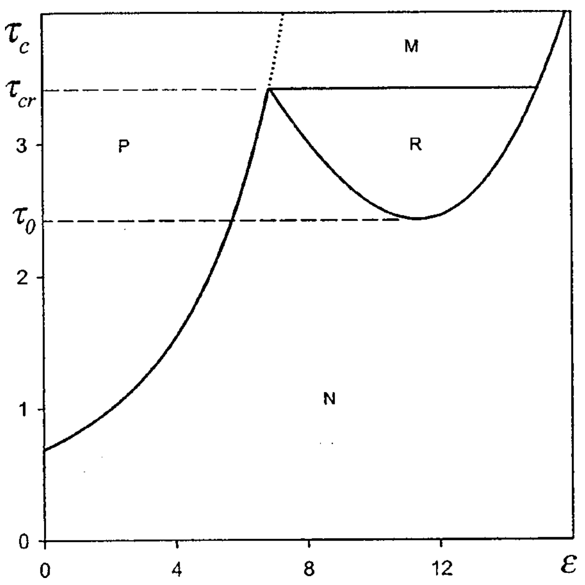

FIG. 2. Phase diagram in the (ε,τ_c) plane at $a=0.8$, $\alpha=0$, $\sigma_a^2=4.84$, $\sigma_m^2=0.01$, $\tau_m=0.01$, $D=1.0$. Domains denoted as P, N, R, and M correspond to positive and negative η values, reentrant transition, and (meta)stable phases, respectively.

where an increase in σ_m^2 transforms the two-connected $\eta(\varepsilon)$ dependence into a one-connected one varying faster. With growth of the time scale τ_c, an increase in the multiplicative noise intensity leads to the shrinking of the metastability domain (see lower panel in Fig. 3). Thus, we can see the dual role of the multiplicative noise: at small intensities $\sigma_m^2 \ll 1$, the main influence is rendered by the cross-correlations between additive and multiplicative components of the noise to sharpen the phase transition (see Fig. 1); on the other hand, a raising of the intensity of the latter component up to $\sigma_m^2 \sim 1$ smears this transition.

According to Fig. 4 only one difference between the phase diagram in the axes (ε,D) and the situation depicted in Fig. 2 takes place. Here, at small magnitudes of the control parameter ε, the domain P of the positive valued order parameter $\eta>0$ relates to the whole region of the coupling parameter D.

To find relations between noise exponent a and the control parameter ε we consider the phase diagram in (ε,a) plane. It appears that for noises with weak cross-correlation $(\tau_c \to 0)$ the new phase arises only at small enough values of a which define the power of the multiplicative noise (see dashed curve in Fig. 5). In other words, considering the class of systems with both additive and multiplicative noises, one should take into account that the ordering processes are possible in the case of weak cross-correlation only if the multiplicative noise has a weak power. For the systems with $a \to 1$ the weak cross-correlation cannot induce new phase formation. According to the solid-dotted curves in Fig. 5, an increase in τ_c leads to appearance of small valued domain of ε where the reorientation transition takes place with a growth. In addition, domains of both positive and negative order parameters, being reoriented, join with the metastable phase region at small values of the index a. However, an increase in a leads to the reentrant phase transition for the

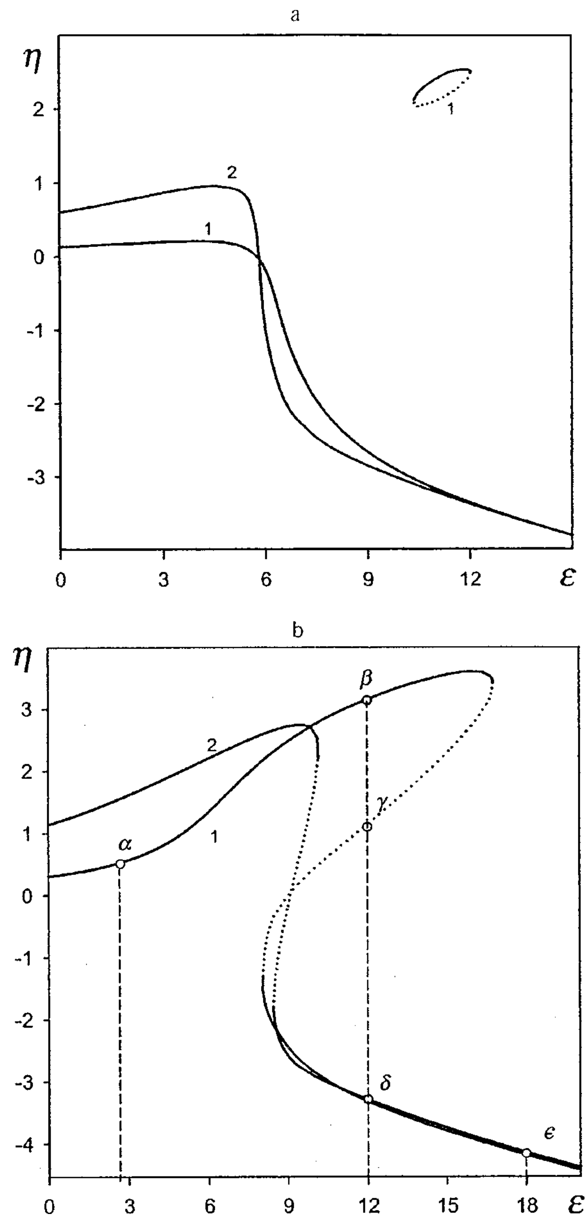

FIG. 3. The order parameter η vs the control parameter ε at $a=0.8$, $\alpha=0.0$, $D=1.0$, $\sigma_a^2=4.84$, $\tau_m=0.01$ and cross-correlation times $\tau_c=2.5$ (a) and $\tau_c=5.0$ (b). Curves 1, 2 relates to the multiplicative noise intensities $\sigma_m^2=0.01$ and $\sigma_m^2=0.25$. Points from α to ϵ address to corresponding curves in Fig. 10.

long range cross-correlations. At small and moderate values of the noise exponent a the system behavior is described by the hysteresis loop formation.

The influence of the damping exponent $\alpha \neq 0$ on the breaking symmetry picture is shown in Fig. 6. It appears that an increase in α transforms the $\eta(\varepsilon)$ dependence in a manner similar to the influence of the cross-correlation time τ_c. Indeed, the passage from the two-connected $\eta(\varepsilon)$ dependence related to curves 1 to one-connected curves 2, 3 can be provided with an increase of both τ_c and α—quite similar to the $\eta(\varepsilon)$ dependencies variation shown in Fig. 1. The conclusion about similarity of the influences of the damping exponent $\alpha \neq 0$ and the cross-correlation time τ_c is confirmed by the

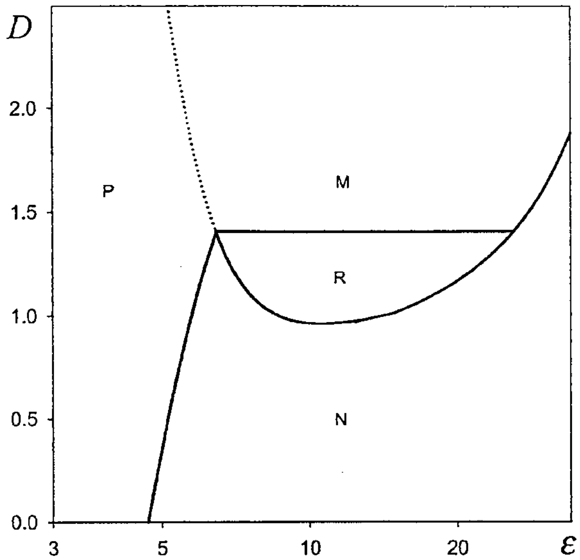

FIG. 4. Phase diagram in (ε, D) plane at $a=0.8$, $\alpha=0$, $\sigma_a^2=4.84$, $\sigma_m^2=0.01$, $\tau_m=0.01$, $\tau_c=2.5$.

phase diagram in plane (ε, α) which is topologically similar to the same in Fig. 2 at large correlation times [see Fig. 7(a)]. According to Fig. 7(b) decreasing the cross-correlation time leads to the reentrant phase transition (within the domain *R*) due to the appearance of the additional region *N* related to the negative values of the order parameter.

Finally, we set up the properties of the Langevin sources which induce ordering processes in the system. To this end, we plot the corresponding phase diagram in (τ_c, a) plane (see Fig. 8). Here, the system undergoes the reorientation transition related to the transformation of the negative valued order parameter into the positive one if τ_c increases. On the other hand, increasing the exponent *a* of multiplicative noise at small ε, we can make the system undergo a chain of phase transitions at which the parameter η changes the sign three times as maximum [at $\varepsilon=6.5$ and $\tau_c=2.85$ for example, see Fig. 8(a)]. The physical situation becomes more simple with an increasing of ε [Fig. 8(b)].

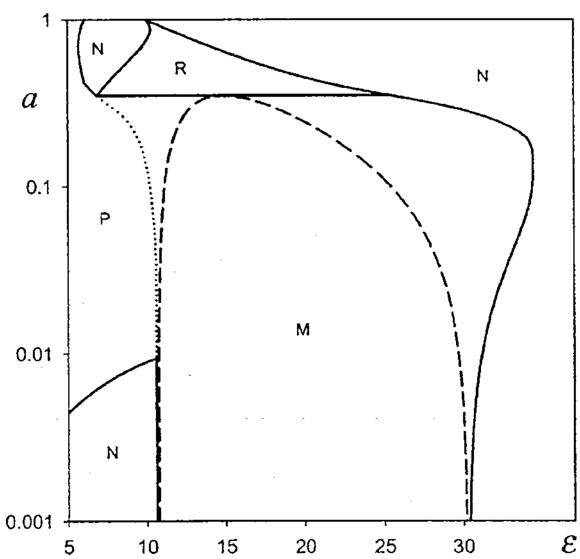

FIG. 5. Phase diagram in (ε, a) plane at $\alpha=0$, $\sigma_a^2=4.84$, $\sigma_m^2=0.01$, $\tau_m=0.01$, $\tau_c=2.5$, $D=1.0$. Dashed curve relates to the limit $\tau_c \to 0$.

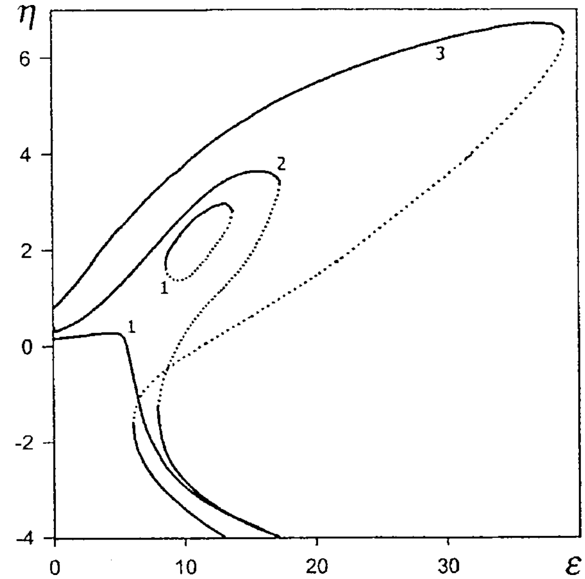

FIG. 6. The order parameter η vs the control parameter ε at $\tau_m=0.01$, $\sigma_a^2=4.84$, $\sigma_m^2=0.01$, $D=1.0$, $a=0.8$: curve 1—$\alpha=0.2$, $\tau_c=2.5$; curve 2—$\alpha=0.2$, $\tau_c=5.0$; curve 3—$\alpha=0.7$, $\tau_c=5.0$.

V. DISCUSSION

To understand the main features of the system under consideration we proceed from the equation of effective motion

$$\dot{x} = \mathcal{D}_1(x) + \sqrt{\mathcal{D}_2(x)}\xi(t) \qquad (50)$$

related to the Fokker-Planck equation (35) where white noise $\xi(t)$ with properties $\langle \xi(t) \rangle = 0$, $\langle \xi(t)\xi(0) \rangle = \delta(t)$ is used. Within the mean-field approach, Eq. (50) takes on the form

$$\dot{\eta} = \langle \mathcal{D}_1(x) \rangle \simeq -\frac{\partial F}{\partial \eta}, \quad F \equiv \delta F(\eta) - h\eta. \qquad (51)$$

Using the definition (48) for the simplest set of indexes $\alpha=0$, $a=1$ a thermodynamic-type potential $\delta F(\eta)$ and a field *h* are defined through the following expressions:

$$\delta F(\eta) = -\frac{\varepsilon + \varepsilon_m}{2}\eta^2 + \frac{1}{4}\eta^4, \quad \varepsilon_m \equiv \tau_m \sigma_m^2, \qquad (52)$$

$$h \equiv \tau_c \sigma_a \sigma_m. \qquad (53)$$

Comparing the first of definitions (52) with the bare potential (44), one can see that the multiplicative noise leads to an increase in the control parameter ε due to the addition ε_m whose magnitude is proportional to the noise intensity σ_m^2 with the coefficient τ_m being self-correlation time. As a result, a growth of the multiplicative noise intensity shown in Fig. 3 causes an increase in the order parameter η at small magnitudes of the control parameter ε. A smearing of the related dependencies $\eta(\varepsilon)$ at moderate values ε is induced by the effective field *h* inherent in the cross-correlation effect

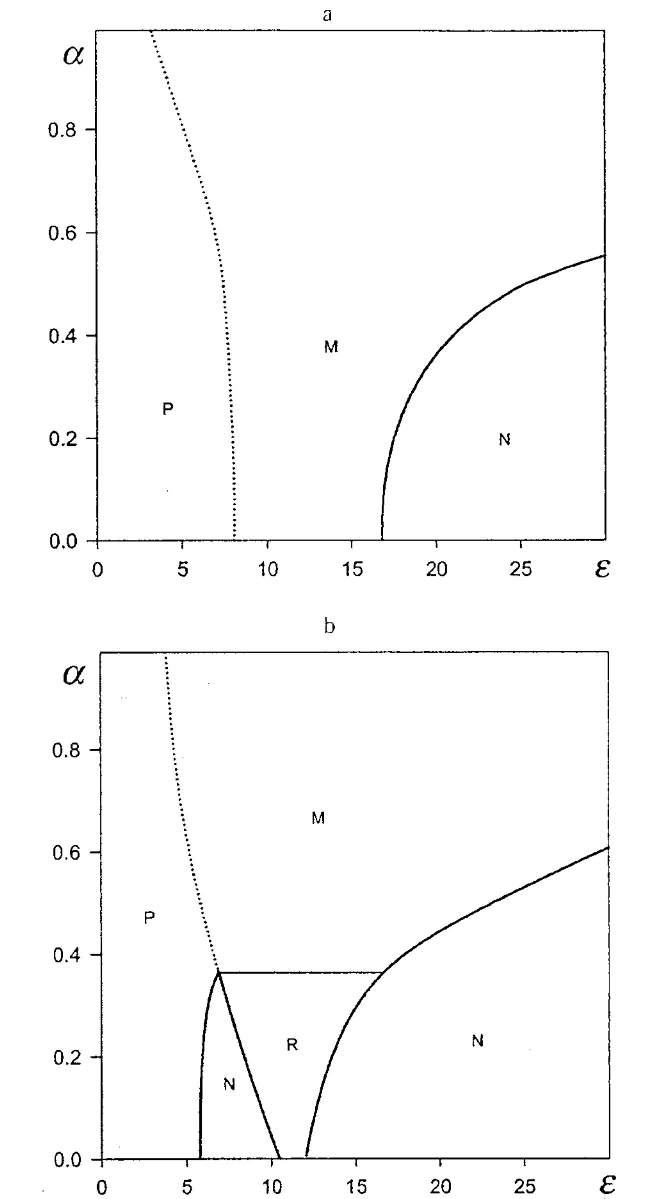

FIG. 7. Phase diagram (ε,α) plane at $\tau_m=0.01$, $\sigma_a^2=4.84$, $\sigma_m^2=0.01$, $D=1.0$, $a=0.8$: (a) $\tau_c=5.0$; (b) $\tau_c=2.5$.

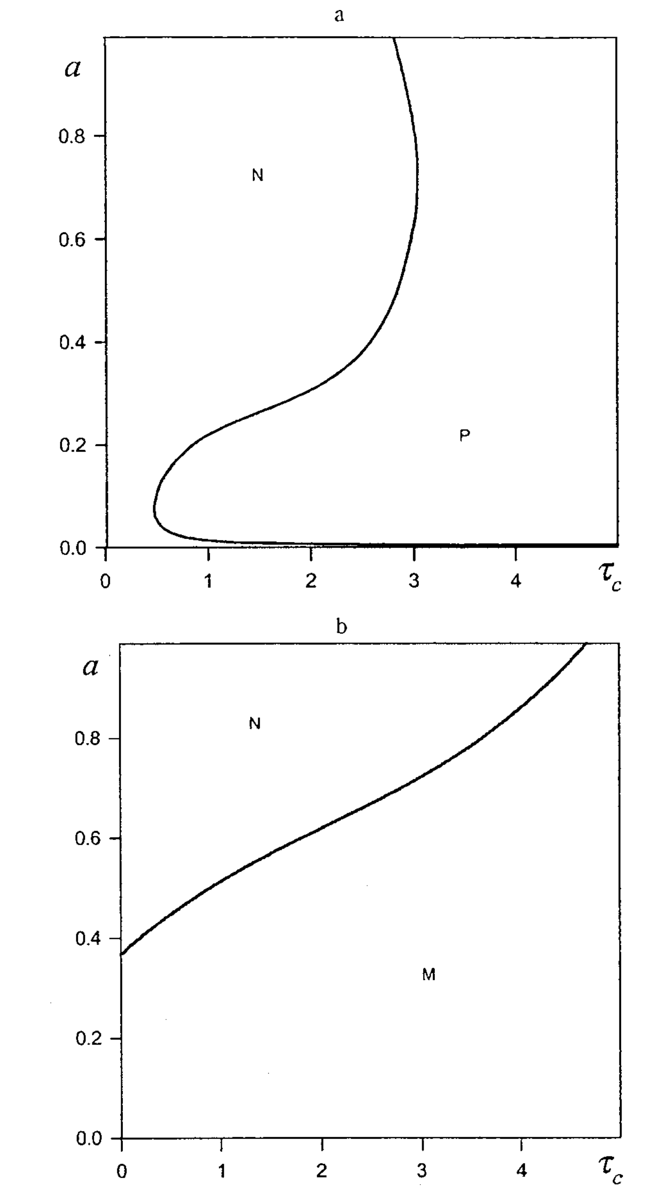

FIG. 8. Noise exponent a vs noise cross-correlation scale τ_c at $\varepsilon=6.5$ (a) and $\varepsilon=15$ (b); other parameters are $\alpha=0$, $\tau_m=0.01$, $\sigma_a^2=4.84$, $\sigma_m^2=0.01$, $D=1.0$.

fixed by the characteristic time τ_c and intensities σ_a^2, σ_m^2. According to Eq. (51) the field h leads to deepening the right minimum of the thermodynamic potential $F(\eta)$. If cross-correlation effects are so slight that the condition

$$\tau_c < C\frac{(\varepsilon+\varepsilon_m)^{3/2}}{\sigma_a\sigma_m}, \quad C \equiv \frac{2}{3^{3/2}} \simeq 0.385 \tag{54}$$

is applied, the field h is less than a critical value $h_c=C(\varepsilon+\varepsilon_m)^{3/2}$ and the right minimum of the thermodynamic potential $F(\eta)$ has a local character. It means that the positive order parameter appears within a two-bounded interval $\varepsilon_c<\varepsilon<\varepsilon^c$ (see curves 2, 3 in Fig. 1). With strengthening cross-correlations, when the condition $h<h_c$ ceases to be valid, a barrier between right and left minima disappears and a domain of the positive order parameter becomes bounded by only the upper boundary ε^c (curves 4, 5 in Fig. 1).

With passage to the general case $a\neq 1$, the thermodynamic potential in Eqs. (51) and (52) takes on the form

$$F = \delta F(\eta) - h\,\mathrm{sgn}(\eta)|\eta|^a, \tag{55}$$

$$\delta F = -\left(\frac{\varepsilon}{2}\eta^2 + \frac{\varepsilon_m}{2}\eta^{2a}\right) + \frac{1}{4}\eta^4 \tag{56}$$

that differs from the initial one by replacement $\eta \to \mathrm{sgn}(\eta)|\eta|^a$. At $a\leqslant 1$, this replacement leads to the more intense variations of the thermodynamic potential $F(\eta)$ within the actual domain $\eta<1$ where the local minima can appear. As a result, a decrease in the index a derives to the metastable phase — in perfect accordance with Fig. 5.

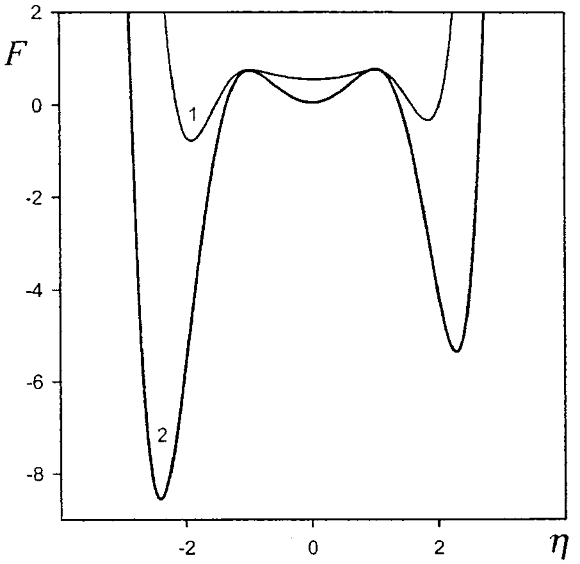

FIG. 9. The form of the thermodynamic potentials given by Eqs. (57)–(61) within the Ito (curve 1) and Stratonovich (curve 2) interpretations.

To ascertain the effect of the index α we consider mean field approach in the extreme case $\alpha=1$, $a=1$. Here, the thermodynamic potential in the equation of motion (51) takes the form

$$F=\widetilde{F}\mp h\eta+\delta F(\eta), \tag{57}$$

where

$$\widetilde{F}\equiv\begin{cases}0, & \text{at } \eta\leqslant -1,\\ (\varepsilon+\varepsilon_m)/2-4h/3-1/6, & \text{at } \eta\in(-1;1],\\ -8h/3, & \text{at } \eta>1,\end{cases} \tag{58}$$

the field h is determined by Eq. (53) and the addition $\delta F(\eta)$ is defined by the expansion

$$\delta F\equiv\frac{A}{2}\eta^2+\frac{B}{3}\eta^3+\frac{C}{4}\eta^4+\frac{E}{5}\eta^5+\frac{G}{6}\eta^6 \tag{59}$$

with the following coefficients:

$$A\equiv\mp(\varepsilon+\varepsilon_m)+2\sigma_a^2,\quad B\equiv(4\pm\tau_c)\sigma_a\sigma_m,$$

$$C\equiv\pm[1+(\varepsilon+\varepsilon_m)]+2(\sigma_m^2-\sigma_a^2),$$

$$E\equiv-4\sigma_a\sigma_m,\quad G\equiv\mp1-2\sigma_m^2; \tag{60}$$

the upper sign in Eqs. (57) and (60) relates to the domain $\eta\in(-1;1]$ and lower sign is taken out of this domain. Comparing these equations with the potential (52) corresponding to the index $\alpha=0$ we find that a spatially dependent damping coefficient (45) leads to the transformation of the second order phase transition into the first order one, as it follows from Figs. 6 and 7.

The form of the thermodynamic potential F given by Eqs. (57)–(60) is shown in Fig. 9 as a function of the order parameter η. It can be seen that such a dependence has three well pronounced minima inherent in the first order phase transition. To pass from the Ito calculus used above to the Stratonovich one, we have to add the term $\frac{1}{2}\mathcal{D}_2'(x)$ to the drift coefficient $\mathcal{D}_1(x)$ in Eq. (51) [4]. Then, the thermodynamic potential $F\equiv-\int\mathcal{D}_1(\eta)d\eta$ has an additional $-\frac{1}{2}\mathcal{D}_2(\eta)$ which transforms the potential given by Eqs. (57)–(60) to the form

$$F=\widetilde{F}-\widetilde{h}\eta+\delta F(\eta), \tag{61}$$

$$\widetilde{F}\equiv\begin{cases}-\sigma_a^2/2, & \text{at } \eta\leqslant -1,\\ (\varepsilon+\varepsilon_m-\sigma_a^2)/2-4h/3-1/6, & \text{at } \eta\in(-1;1],\\ -8h/3-\sigma_a^2/2, & \text{at } \eta>1,\end{cases}$$

$$\widetilde{h}\equiv(1\pm\tau_c)\sigma_a\sigma_m,$$

$$\delta F\equiv\frac{A}{2}\eta^2+\frac{B}{3}\eta^3+\frac{C}{4}\eta^4+\frac{E}{5}\eta^5+\frac{G}{6}\eta^6,$$

$$A\equiv\mp\varepsilon+[4\sigma_a^2-(1\pm\tau_m)\sigma_m^2],$$

$$B\equiv(10\pm\tau_c)\sigma_a\sigma_m,$$

$$C\equiv\pm(1+\varepsilon)-[4\sigma_a^2-(6\pm\tau_m)\sigma_m^2],$$

$$E\equiv-9\sigma_a\sigma_m,\quad G\equiv\mp1-5\sigma_m^2, \tag{62}$$

where we take into account Eq. (49) at $\alpha=1$, $a=1$. Comparison of the dependencies (57) and (61) given in Fig. 9 shows that the Stratonovich addition promotes to strengthening of the first order transition.

We proceed with the consideration of the form of the probability distribution function (40) that is responsible for the reorientation transition related to curve 1 in Fig. 3(b). As can be seen comparing the curves α and ϵ depicted in Fig. 10(a), the positive magnitudes of the order parameter $\eta>0$ is related to the distribution whose right maximum has a larger height and is wider than the left one (and vice versa at $\eta<0$). A much more complicated picture takes place with growth of the correlation time τ_c when strongly pronounced maximum of the distribution (40) is transformed from the left into the right one by means of passage via the bimodal dependence [see curves β, γ, and δ in Fig. 10(b)].

The situation considered above corresponds to a constant damping coefficient (45) when the distribution (40) has smooth form due to the index $\alpha=0$. In the general case $\alpha\neq0$, the distribution $\mathcal{P}_\eta(x)$ exhibits a pair of strong maxima at symmetrical points $x=\pm1$ (see Fig. 11). The analytical form of these maxima follows from the estimations

$$\mathcal{D}_1(x)\simeq2\alpha(\sigma_a\pm\sigma_m)^2x(x^2-1)^{2\alpha-1},$$

$$\mathcal{D}_2(x)\simeq(\sigma_a\pm\sigma_m)^2|x^2-1|^{2\alpha} \tag{63}$$

that are given by the dependencies (48) and (49) near the points $x=\pm1$. As a result, we arrive to the integrable singularities

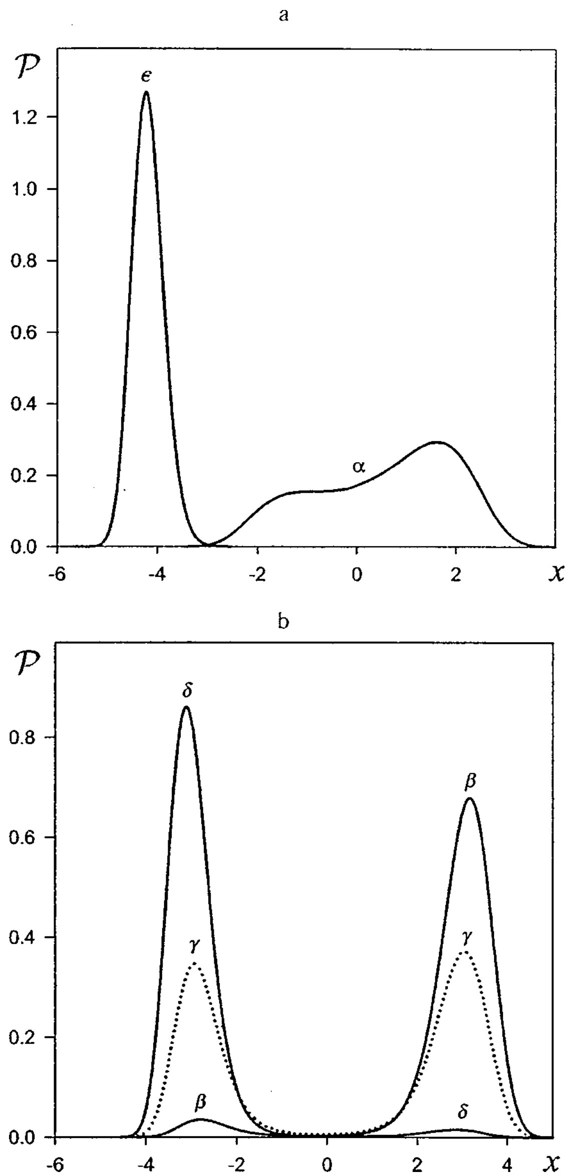

FIG. 10. Probability distributions addressed to the different points of the dependence $\eta(\varepsilon)$ related to curve 1 in Fig. 3(b): curves α, ϵ in panel (a) correspond to the values $\varepsilon=3$, $\eta=0.56$ and $\varepsilon=18$, $\eta=-4.18$, respectively; in panel (b) the control parameter is equal $\varepsilon=12$ and curves β, γ, δ correspond to different magnitudes of the order parameter $\eta=3.15$, $\eta=1.14$, and $\eta=-3.32$.

$$\mathcal{P}_\eta(x) \simeq \frac{\mathcal{Z}_\eta^{-1}}{(\sigma_a \pm \sigma_m)^2 |x^2-1|^\alpha} \tag{64}$$

which have the form of the maxima shown in Fig. 11. In contrary, near the point $x=0$ one has the estimations

$$\mathcal{D}_1(x) \simeq a\sigma_a\sigma_m\tau_c|x|^{a-1}, \quad \mathcal{D}_2(x) \simeq \sigma_a^2 \tag{65}$$

which lead to the expression

$$\mathcal{P}_\eta(x) \simeq \frac{\mathcal{Z}_\eta^{-1}}{\sigma_a^2}\exp\left(\frac{\sigma_m}{\sigma_a}\tau_c|x|^a\right) \simeq \frac{\mathcal{Z}_\eta^{-1}}{\sigma_a^2}. \tag{66}$$

Thus, the singularities of the drift coefficient $\mathcal{D}_1(x)$ at the point of origin has an integrable character that results in the finite value of the probability distribution function.

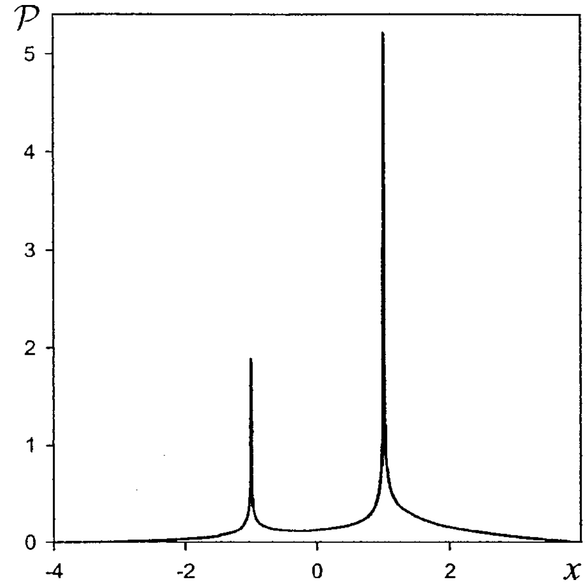

FIG. 11. Probability distribution related at $a=0.5$, $\alpha=0.5$, $\varepsilon=2$.

Traditionally, one presents the distribution function (40) in the Boltzmann-Gibbs exponential form

$$\mathcal{P}(x) \equiv \exp\left\{-\frac{V_{EF}(x)}{\sigma_a^2}\right\} \tag{67}$$

where an effective potential

$$V_{EF} \equiv -\sigma_a^2\int\frac{\mathcal{D}_1(x)}{\mathcal{D}_2(x)}dx + \sigma_a^2\ln\mathcal{D}_2(x) \tag{68}$$

is introduced which is governed by the probability distribution in the usual manner [1]. Usage of the definitions (48) and (49) in the simplest case $a=1$, $\alpha=0$ leads to explicit form of the potential (68):

$$V_{EF} = -\mathcal{H}x + \mathcal{V}(x),\ \mathcal{H} \equiv \mathcal{D}\eta + (\tau_c - 2)\sigma_a\sigma_m,$$

$$\mathcal{V} \equiv \frac{\mathcal{A}}{2}x^2 + \frac{\mathcal{B}}{3}x^3 + \frac{\mathcal{C}}{4}x^4,$$

$$\mathcal{A} \equiv (D-\varepsilon) - (2+\tau_m)\sigma_m^2 + 2\frac{\sigma_m}{\sigma_a}(D\eta + \tau_c\sigma_a\sigma_m),$$

$$\mathcal{B} \equiv 2\frac{\sigma_m}{\sigma_a}[(\varepsilon - D) + (1+\tau_m)\sigma_m^2] - 3\left(\frac{\sigma_m}{\sigma_a}\right)^2(D\eta + \tau_c\sigma_a\sigma_m),$$

$$\mathcal{C} \equiv 1 - \left(\frac{\sigma_m}{\sigma_a}\right)^2[3(\varepsilon - D) + (2+3\tau_m)\sigma_m^2] + 4\left(\frac{\sigma_m}{\sigma_a}\right)^3(D\eta + \tau_c\sigma_a\sigma_m), \tag{69}$$

where we kept only the terms up to the fourth order in the stochastic variable x. The form of the dependence (68) is depicted in Fig. 12 for different sets of the indexes a and α.

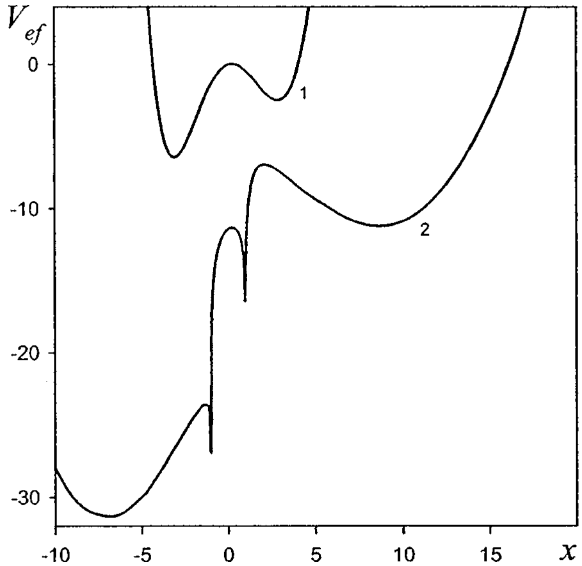

FIG. 12. The form of the effective potential (68): curves 1 and 2 correspond to the cases $a=1$, $\alpha=0$ ($\varepsilon=10, \eta=-2.8$) and $a=1$, $\alpha=1$ ($\varepsilon=10, \eta=-4.88$), respectively.

Comparing respective curves, we conclude that the growth of the index α promotes a strengthening of the strong minima at points $\eta=\pm 1$ of the $V_{EF}(x)$.

VI. CONCLUSIONS

In this paper we have considered the effect of the ordering of stochastic system with two correlated noises. Doing so, we have used the model of a system with Landau-like potential $V_0(x)$, subject to both additive and multiplicative noises with amplitude of the latter in the form of the power-law function $|x|^a$, $a\in[0,1]$ and affected by x-dependent damping with coefficient $\gamma(x)=|x^2-1|^{-\alpha}$, $\alpha\in[0,1]$. Within the framework of both the cumulant expansion method and mean-field theory, the stationary picture of the ordered states is investigated in detail. We have shown that the fluctuation cross-correlations can lead to the symmetry breaking of the distribution function even in the case of the zero-dimensional system. When introducing the spatial coupling, noise cross-correlations can induce phase transitions where the order parameter $\eta=\langle x\rangle$ varies discontinuously or in a reentrant manner. We have studied the specific intervals of the magnitudes of system parameters where the ordered phase can be formed. To this end, the principal phase diagrams are obtained to illustrate the role of the multiplicative noise exponent a, spectral characteristics of fluctuations (autocorrelation time τ_m and cross-correlation time τ_c), amplitudes of both additive σ_a and multiplicative σ_m noises, exponent α of the kinetic coefficient $\gamma(x)$, as well as deterministic parameters (dimensionless temperature ε and intensity of the spatial coupling D).

The picture studied above allows us to generalize the theory of phase transitions for systems with a set of stochastic forces of different nature. Based on the mean-field approach, we show that the system can be described through a thermodynamic potential $F(\eta)$ whose construction differs principally from the bare potential $V_0(x)$: so, if the latter has the simplest x^4 form, the former is shown to be of the η^6 form. Coefficients of related expansion are obtained to define terms of the even powers through the dimensionless temperature ε, intensity of spatial coupling D, autocorrelation time τ_m and intensities of both additive σ_a^2 and multiplicative σ_m^2 noises; terms of the odd powers are defined through the characteristics τ_c, σ_a, and σ_m of the noise cross-correlations, respectively. Thus, we can conclude that the phase transition tends to transform its character from a continuous to a discontinuous one due to the noise cross-correlation strengthening. This trend is displayed more strongly with the growth of the index α whose value determines the x dependence of the damping coefficient $\gamma(x)$. On the other hand, transition from the Ito calculus to the Stratonovich one promotes strengthening of the discontinuous transition.

Obtained results can be applied to complex systems which are far-off-equilibrium and hold several collective degrees of freedom. As it is shown in the consideration of a three-dimensional Lorentz-like system with noises being initially additive in nature, a usage of the saving principle reduces two of these noises to multiplicative ones [18]. The physical reason of such a picture is hierarchical subordination of different degrees of freedom. According to our previous considerations [19,20] a typical example of such a type takes place in solid state physics where a reentrant metastable phase can appear if the matrix phase relates to random ensemble of defects of different dimensions subject to the field of plastic flow (driven-dislocation-vacancy-ensemble). Here, in the course of plastic flow different defect structures alternate according to the picture of a first-order phase transition. Moreover, the structural reorientation transitions take place where the sign of the order parameter is related to the resulting direction of the Burgers vectors of dislocation cluster. One more example of the above studied behavior is given by the reentrant glass transition in colloid-polymer mixtures [21].

Finally we note that all the presented results have been derived for a system with a nonconserved order parameter. The perspective of further exploration is to investigate the system with a conserved order parameter.

ACKNOWLEDGMENTS

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